THE PARAMAGNETIC ELECTRON RING

Winfried Herrmann

IPP 1/253 July 1990

MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

8046 GARCHING BEI MÜNCHEN

MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

GARCHING BEI MÜNCHEN

THE PARAMAGNETIC ELECTRON RING

Winfried Herrmann

IPP 1/253 July 1990

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

The Paramagnetic Electron Ring

Winfried Herrmann

Max-Planck-Institut für Plasmaphysik, D-8046 Garching, FRG
EURATOM Association

Abstract

The axial acceleration of electron rings is investigated in the presence of an azimuthal field B_φ for the pure magnetic acceleration in an expanding magnetic field. It is shown that, with the B_φ-field present the expanding field is replaced by a cusp field, the full electron energy which is originally stored in the transverse motion of the electrons can be converted to energy of axial motion.

Introduction

Electron rings have been proposed [1] and investigated [2] as a vehicle for acceleration of heavy ions. In the simplest form of an electron ring accelerator, the original transverse energy of the relativistic electrons is transformed into axial energy in an expanding magnetic field. In most experiments that have been performed following this concept electron rings were formed at modest energies at large radii in low fields and were brought to their final state with high energy and small radius, suitable for acceleration, by compression in a mirror type magnetic field. During compression and in the compressed state during the shift of the ring from the mirror field to the expanding field the ring crossed dangerous betatron resonances that could enlarge the minor radius of the electron ring and thus reduce the internal electric field, (the holding power of the ring), which should be maximized for a good performance of an accelerator.

One proposal to avoid the resonances and their deleterious influence was to apply an azimuthal magnetic field B_φ, in addition to the expanding field. One of the first to show the beneficial consequences of this proposal was A. Schlüter [3] (Lecture given in IPP, Garching, in 1970 unpublished). He showed that main resonances could be avoided and effects on acceleration seemed to be tolerable. This proposal was investigated in more detail by P. Merkel [4] (IPP Report IPP 0/4, March 1971). With regard to acceleration the result of his paper was, that the B_φ-field "reduces acceleration, just as if the mass of the ring were increased by a factor of $1 + \alpha^2$, where $\alpha = B_\varphi/B_z$ is the ratio of the B_φ-field to the main field B_z" at the starting point of the expansion acceleration. As long as this linear theory was applicable this reduction of acceleration was of no concern because it could easily be compensated by a faster expanding field. What however could be the effect on a real accelerator? Would the increase of the ring mass also exist in the nonlinear regime and would this mean, that the final axial energy in the expanding field would be reduced, e.g. by a factor of 2, if α was chosen to be 1? How would the ring behave - or at least the single electrons in the ring -, when the ring - or the single electrons -, reaches the area, where the expanding field has decreased to zero value?

Answers to these questions were sought with the help of computer calculations that the author performed during his visit to the electron ring group in the Lawrence-Berkeley-Laboratory in 1972. The results of these calculations were summarized in a comment in an internal report [5] (ERAN-204, Lawrence-Berkeley-Laboratory, Berkeley, October 1972) that, in its main part, deals with a different subject (the nonexistance of an instability that had been predicted).

This paper briefly summarizes the effects of the B_φ-field on the particle motion in linear approximation, following the paper by P. Merkel. It then outlines the main components of a computer programme for calculation of the particle orbits and finally describes and discusses the somewhat surprising result.

Ring acceleration in an expanding field with superimposed B_φ-field

An expanding field obeying Maxwell's equations can be represented in cylindrical geometry by the following two equations

$$B_r = B_0 \cdot \epsilon \cdot \frac{r}{2}$$

$$B_z = B_0(1 - \epsilon \cdot z)$$

The azimuthal field is assumed to be produced by a current, flowing in a rod on the cylinder axis:

$$B_\varphi = \frac{B_{\varphi 0} \cdot R_0}{r}$$

Here ϵ is a measure of the non-uniformity of the field, B_0 is the value of the axial field at z=0, and $B_{\varphi 0}$ is the value of the azimuthal field at the ring position R_0 for z=0.

Choosing suitable initial conditions and neglecting radial velocities and acceleration Merkel solves the equation of motion and finds for the axial velocity, v_z, as a function of z for small values of $\epsilon \cdot z$ an equation which can be written in the following form:

$$E_z = \frac{m}{2} \cdot v_z^2 = \frac{1}{2} \frac{m}{1 + \alpha^2} \cdot v_{\varphi 0}^2 \cdot \varepsilon \cdot z$$

This equation implies that the energy of the axial motion increases linearly with the motion in the expanding field. The energy at a certain point, that is for a certain reduction of the magnetic field decreases in inverse proportion to $1 + \alpha^2$.

From this equation it follows that for a pure expanding field accelerator the beneficial action of the superimposed B_φ- field for stabilizing the ring against betatron oscillations has to be paid for by a loss in gain.

But apart from the regrettable loss of efficiency of an expanding field accelerator it seems to be an interesting question how far the ring follows the linear assumptions and how the motion of the particles looks like in the area of the vanishing field. The following calculations will show that a consequent application of the expanding field method can avoid the loss in acceleration energy and also show how the particle motion is affected.

Calculation of particle motion

For the calculation of the particle motion the full relativistic equations of motion have been used in cylindrical geometry with the components r, φ and z. The full set of equations has been used unlike in the analytic calculation of Merkel and the energy equation has been used only to check the accuracy of the calculations, as in the case without electric field the energy of the particle should not change.

For the components of the magnetic field the expressions given in the last chapter have been used. The electric field is zero. For the main case of interest here the following initial conditions have been chosen:

$$\gamma = 20$$

$$r = \frac{m_e \cdot \gamma \cdot v_{\varphi 0}}{e \cdot B_{zo}}$$

$$\varphi = z = v_z = v_r = 0$$

$$B_{zo} = 2\ [T]$$

$$\varepsilon = 1\ [m^{-1}]$$

$$\alpha = +1, 0, -1$$

With the value of $\varepsilon = 1$ the axial magnetic field B_z is zero at z = 1.0 m.

For the content of this paper it is not at all necessary that the electrons are relativistic. Similar calculations, not reported here, have been performed for low energy electrons of 5 keV ($\gamma = 1.01$) which give the same results. The initial and general conditions are chosen here in accordance with the conditions of the electron ring accelerator as the starting point of this discussion.

Comparison with Merkel's calculations

Merkel's calculations of the ring acceleration are only valid for $\varepsilon z \ll 1$, but as Fig. 1 shows, the differences between the analytical and the computer calculated quantities are not too large even with εz approaching unity. In Fig. 1 a few quantities are plotted as a function of the axial dimension z for different values of α. As one expects, the originally transverse energy of the ring is almost fully transformed into longitudinal energy for $\alpha = 0$ at z = 1 m, where B_z goes to zero. The small difference between v_z and $v_{\varphi o}$ is due to radial velocity which is connected with the radial expansion of the ring. Fig. 2a shows the radial motion as a function of the azimuthal angle for the ring motion between z = 0 m and z = 1 m.

The axial velocity v_z for $\alpha = +1$ or $\alpha = -1$ is very similar during the initial phase and corresponds well to the value calculated by Merkel (dashed line). Approaching z = 1m the value of v_z for $\alpha = -1$ stays slightly below the value which would follow from the application of Merkel's calculation and would be given by: $v_z(\alpha = -1) = \frac{v_z(\alpha=0)}{\sqrt{1+\alpha^2}}$. The projection of the particle orbit onto the plane perpendicular to the axis is given in

Fig. 2c. The particle spirals about the axis with decreasing radius. The ratio of the axial velocities δ_α for $\alpha = 0$ and $\alpha = -1$ is plotted at the bottom of Fig. 1. The value for α = -1 is always below the limiting value of Merkel and shows that acceleration is decreased further (the "mass" of the particles is increased further) than expected from linear theory. For $\alpha = +1$ the deviations at small values of z from the Merkel value tend to be smaller than for $\alpha = -1$. However, if z approaches 1 m v_φ drops drastically. As can be seen from Fig. 2b this is due to the large radial velocity: the ring expands faster for $\alpha = +1$ than for $\alpha = 0$ (Fig. 2a). Following the increase in v_r, v_z drops also, when z = 1 is approached.

The most remarkable point in Fig. 1 is the fact that the azimuthal velocity v_φ is still large at z = 1m where B_z becomes zero for $\alpha = -1$. On the one hand this means that indeed only part of the original azimuthal energy of the ring can be used for acceleration (even less than was expected from the linear calculation of Merkel) but on the other hand the question arises of what will happen to the azimuthal velocity when B_z remains zero beyond z = 1 m or if it even changes sign.

The "paramagnetic" motion of the ring

To investigate the question posed in the last section the calculations of the particle motion were continued beyond z = 1m. For the magnetic fields chosen B_r continues with the same function of r and does not change the sign. B_z changes the sign and its absolute value increases with z. Topologically the arrangement has been changed from a single expanding coil to a cusp field with two coils, the axial field directions of which are opposite.

In Fig. 3 the results are plotted for an acceleration length of 3m. Shown are the values of B_φ and B_z at the position of the electron and its velocities v_φ and v_z. The most important result is that v_z is not limited now to the value $\delta_\alpha = \frac{v_{\varphi 0}}{\sqrt{1+\alpha^2}}$ found when considering the pure expanding field, but continues to increase and eventually approaches $v_{\varphi 0}$. v_φ does not change sign when B_z does. The particle continues to encircle the axis in the same direction as it did before the axial field changed its sign. The electrons, the motion of which is generally diamagnetic in a given field, now behave in a paramagnetic manner with respect to the axial field. As the plot shows the effect is not small. The electron that starts diamagnetically in a field of 2 T is found in paramagnetic motion at z = 3m in an axial field of 4 T. The azimuthal field is of the same order in this case. It does not change the sign at z = 1m and is the dominant field there.

The explanation for this particle behaviour is found in the force equations which read as follows:

$$Fr = e(v_\varphi \cdot B_z - v_z B_\varphi)$$

$$F\varphi = e(v_z \cdot B_r - v_r \cdot B_z)$$

$$Fz = e(v_r B_\varphi - v_\varphi B_r)$$

Let us first discuss the last equation. The accelerating force does not depend on B_z and its sign. For negative B_z and B_r and positive $v_{\varphi 0}$ the acceleration of the particle is in positive direction v (e has been taken positive). As the figures 2 show the term $v_r \cdot B_\varphi$ is always negative and the accelerating force is always reduced. The acceleration, however, stays positive as long as the first term is smaller than the second. This apparently is always the case for $\alpha = -1$. For $\alpha = +1$ Fig. 1 shows a deceleration when the particle approaches the zero point of the axial field. This is caused by the large radial velocity at the end of the expansion which makes the first term in this equation larger than the second one.

We now consider the radial force. The second term here depends on the sign of B_φ. If α is positive and hence B_φ negative (B_r in this discussion is always negative) the radial force is reduced and the particle radius will be enlarged. If α is positive and B_φ is negative both terms in the equation add to the force. If the term $v_\varphi \cdot B_z$ reduces during the acceleration expansion this can be compensated by the growing term $v_z \cdot B_\varphi$. Even if B_z is zero the radial force is not, as $v_z \cdot B_\varphi$ zero. The term $v_z \cdot B_\varphi$ can indeed force the electron to rotate in the opposite sense to the case when only the first term $v_\varphi \cdot B_z$ is present.

The actual motion of the electrons is certainly influenced by the choice of B_φ; its radial dependence, especially the dependence of the radial velocity on z can be varied drastically, even the sign of the radial motion can be changed. This, however, does not influence the "paramagnetic" motion of the particles and the possibility to convert the rotational energy to axial energy. Calculations have been performed for low energy particles in B_φ-fields that increase with the radius like in tokamaks with constant current density. The results are qualitatively the same as the ones discussed here.

Summary and discussion

The advantage of avoiding betatron resonances by the application of a B_φ-field does not have to be paid for (at least in principle, as the stability of the ring has not been discussed here) by a reduction in acceleration gain. With the correct choice of the sign of the azimuthal field all of the original transverse energy can be converted to longitudinal energy, if a cusp field is applied instead of a simple expanding field. For full conversion the fields, however, approach infinity. The longitudinal motion of the electrons in the azimuthal B_φ-field assures a balancing force to the centripetal force such that the electrons continue to rotate in the same sense in the inverse axial field as they did in the original field. With respect to the axial field the electrons are forced to behave in a paramagnetic manner.

Their rotation frequency around the axis can be expressed in the following form (neglecting a radial velocity):

$$\omega_c = +\omega_z - \frac{v_z}{v_\varphi} \cdot \omega_\varphi = +\omega_z (1 - \frac{v_z}{v_\varphi} \cdot \frac{\omega_\varphi}{\omega_z})$$

where ω_z and ω_φ are the cyclotron frequencies with the proper sign in the axial and the azimuthal fields respectively. v_r stays small only in the case with negative α. In this case $B_\varphi > 0$ and v_z and v_φ are both positive. B_z starts with negative values and goes through zero to positive values. As long as B_z is negative the diamagnetic effect is enhanced but becomes paramagnetic if B_z changes sign.

A real application of this effect in an electron ring accelerator would depend not only on the question of ring stability but also on the practical availability of magnetic fields that are rather large for a high conversion of transverse into longitudinal energy for a large value of α.

Whether the effect described here has some application in other fields than the electron ring accelerator has not been investigated in detail. In plasma devices like tokamaks the class of particles described here - particles encircling the axis - is usually very small. On the other hand the effect does not depend on relativistic energies nor on the presence of expanding fields. If instead of expanding fields fields are chosen that are constant along the axis and acceleration of the particles is accomplished by electric fields, similar to the case of Ohmic heating in tokamaks, particles that without electric field encircle the axis continue to do so with axial acceleration with a cyclotron frequency that follows the formula given above. An influence of this effect might be found for special types of particles in a plasma like run-away electrons or charged fusion products or for a toroidally rotating plasma.

Acknowledgement:

This paper was written on the occasion of the retirement of Prof. A. Schlüter, whose influence on this work is gratefully acknowledged. I also thank my colleagues Dr. W. Dommaschk, Dr. P. Merkel and Dr. U. Schumacher for suggestions and discussions.

References

[1] V.I. Veksler et al., Atomnaja Energija NS (1957)

[2] See for example: U. Schumacher et al., Phys. Lett. **51A**, 367 (1975)

[3] A. Schlüter, Lecture given in IPP, Garching, (Sept. 1969), unpublished

[4] P. Merkel, IPP Report, IPP 0/4, Garching, March 1971

[5] W. Herrmann, LBL-Report ERAN-204, Berkeley, October 1972

Figure Captions

Fig. 1 Azimuthal and axial velocities v_φ and v_z as a function of distance z along the expanding field for different values of $\alpha = B_{\varphi 0}/B_{zo}$. Dashed curves: values applying Merkel's (linear) theory. Also given are the axial field at ring position and for $\alpha = -1$ the ratio of the axial ring velocity to the ring velocity for $\alpha = 0$.

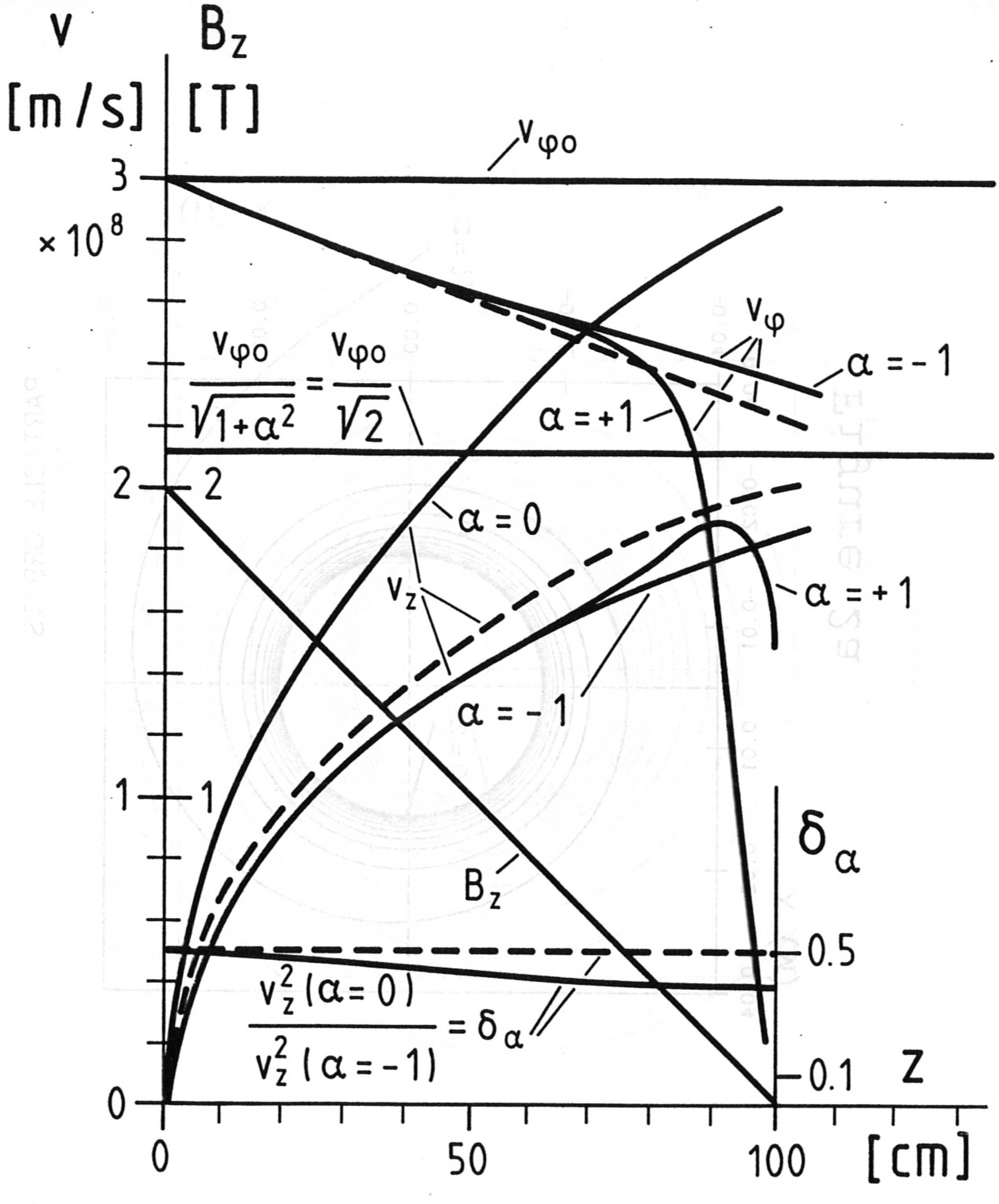
Fig. 2 Projection of the particle motion onto a plane perpendicular to the axis.

a) : for $\alpha = 0$

b) : for $\alpha = +1$

c) : for α = -1

Fig. 3 v_φ and v_z for $\alpha = -1$ in the cusp field arrangement as a function of distance z along the accelerating field. Also B_φ and B_z are given at ring position.



v
B_z
[m/s]
[T]
$v_{\varphi o}$
3
$\times 10^8$
v_φ
$\alpha = -1$
$\frac{v_{\varphi o}}{\sqrt{1+\alpha^2}} = \frac{v_{\varphi o}}{\sqrt{2}}$
$\alpha = +1$
2
2
$\alpha = 0$
v_z
$\alpha = +1$
$\alpha = -1$
1
1
δ_α
B_z
0.5
$\frac{v_z^2(\alpha = 0)}{v_z^2(\alpha = -1)} = \delta_\alpha$
0.1
z
0
0
50
100
[cm]

Figure 1

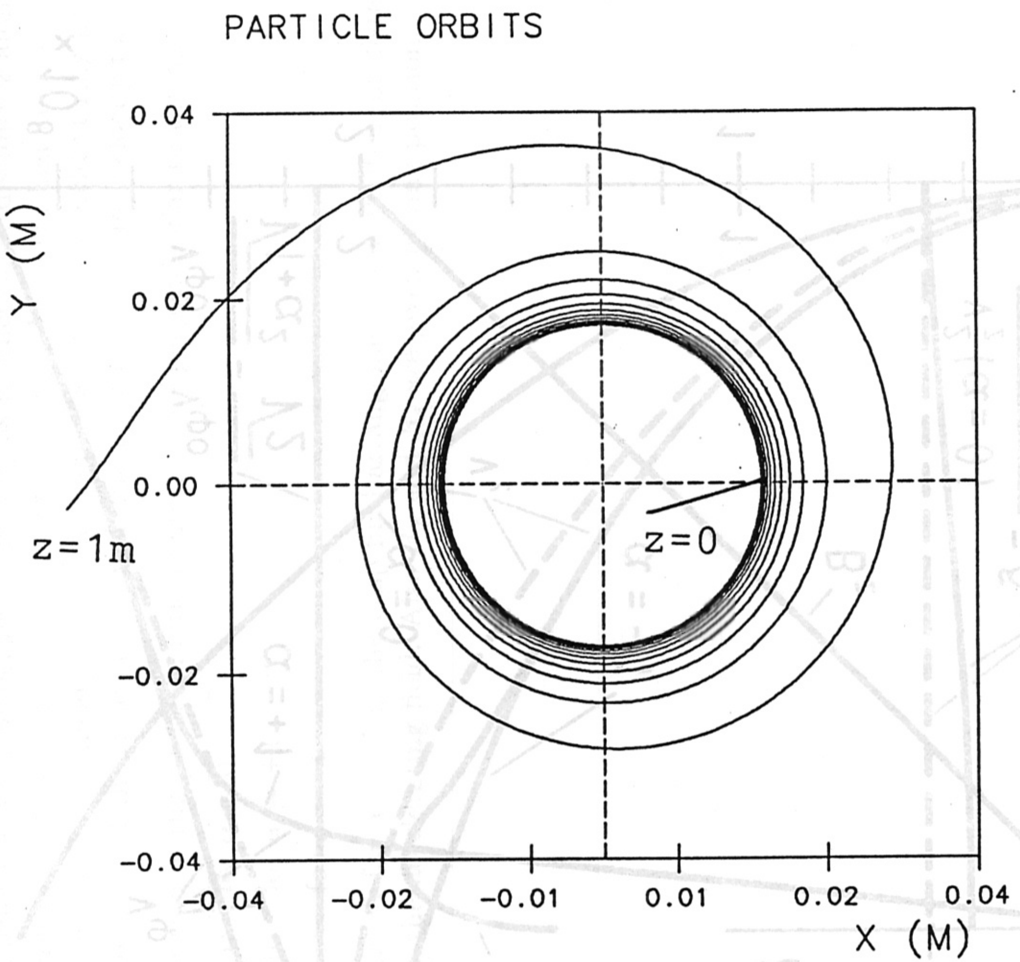

PARTICLE ORBITS
Y (M)
0.04
0.02
0.00
-0.02
-0.04
-0.04
-0.02
-0.01
0.01
0.02
0.04
X (M)
z=1m
z=0

Figure 2a

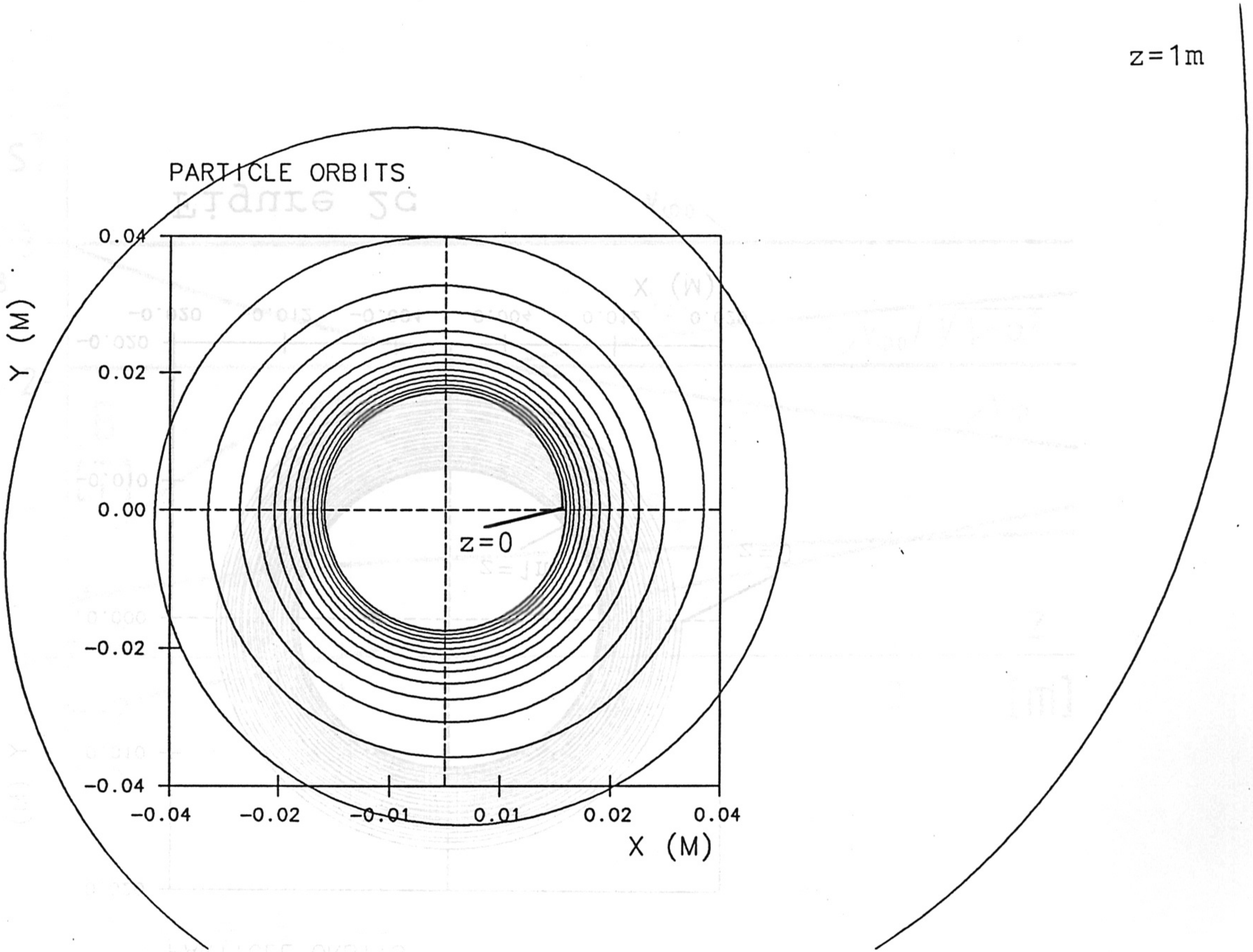
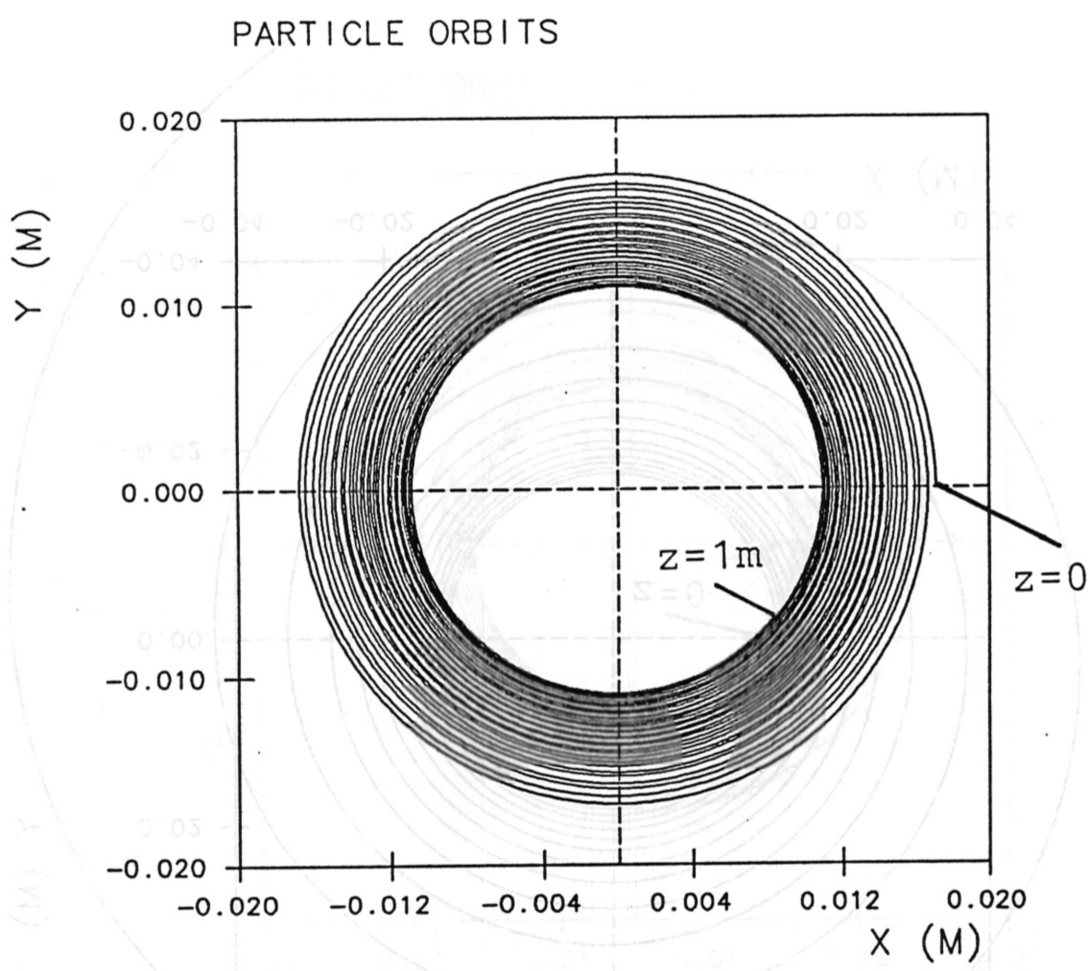
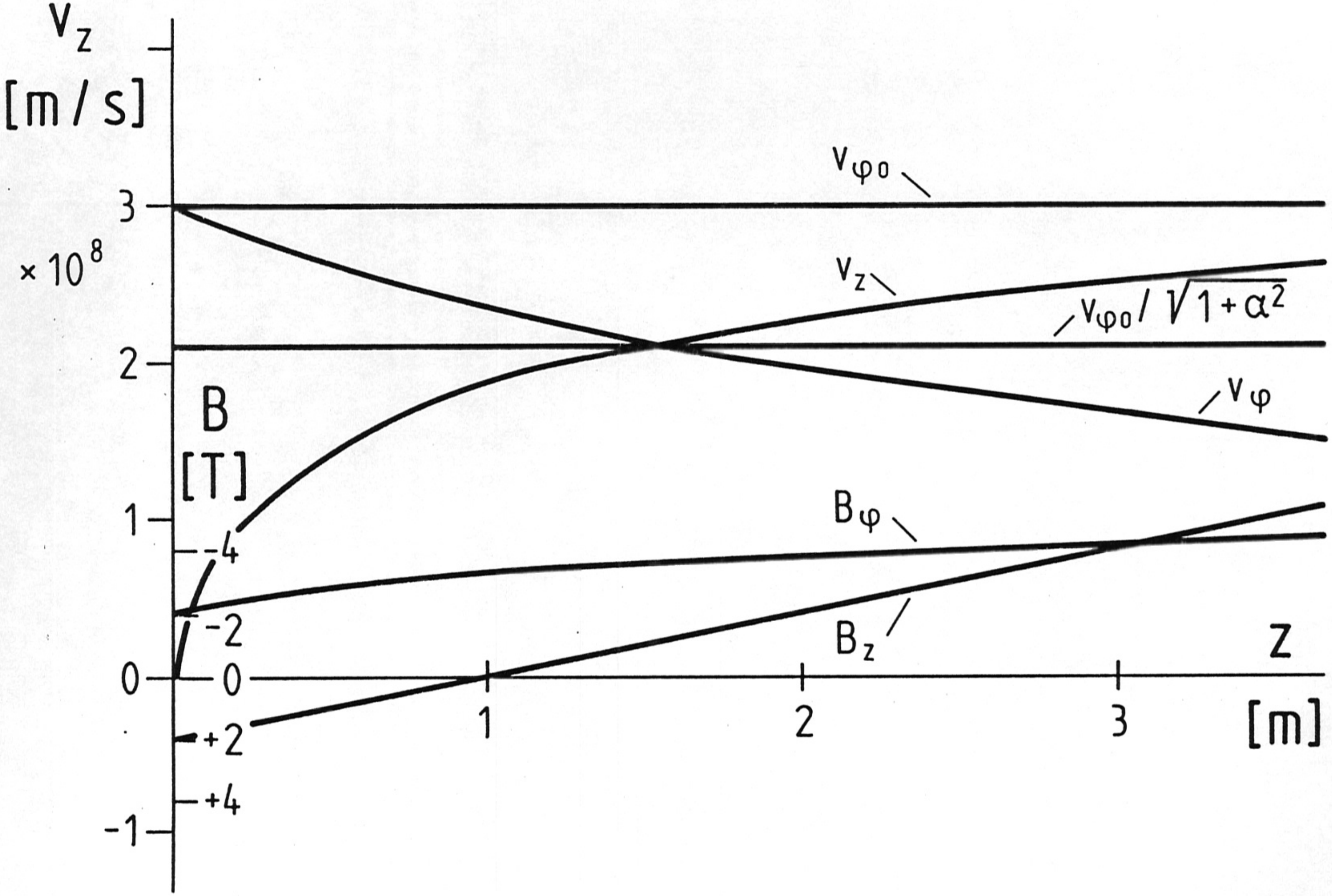

PARTICLE ORBITS
Y (M)
0.04
0.02
0.00
-0.02
-0.04
-0.04
-0.02
-0.01
0.01
0.02
0.04
X (M)
z=0
z=1m

Figure 2b



PARTICLE ORBITS
Y (M)
0.020
0.010
0.000
-0.010
-0.020
-0.020
-0.012
-0.004
0.004
0.012
0.020
X (M)
z=1m
z=0

Figure 2c



v_z
[m / s]
$\times 10^8$
3
2
1
0
-1
B
[T]
-4
-2
0
+2
+4
$v_{\varphi 0}$
v_z
$v_{\varphi 0} / \sqrt{1+\alpha^2}$
v_φ
B_φ
B_z
z
[m]
1
2
3

Figure 3